

# Outline of “Shear Capacity of PC Beams Based on Beam and Arch Actions”

## 「ビーム・アーチ機構を考慮した PC 梁のせん断耐力」の概要



\* Devin GUNAWAN: Tokyo Institute of Technology (Currently, Kajima Corporation)

Devin GUNAWAN : 東京工業大学 (現 鹿島建設 (株))

\*\* Kazumasa OKUBO, Dr.Eng., P.E.Jp: Tokyo Institute of Technology (Currently, Kajima Corporation)

大窪 一正, 博士 (工学), 技術士 (建設部門) : 東京工業大学 (現 鹿島建設 (株))

\*\*\* Takuro NAKAMURA, Ph.D.: Civil Engineering Research Institute for Cold Region

中村 拓郎, 博士 (工学) : (国研) 土木研究所 寒地土木研究所

\*\*\*\* Junichiro NIWA, Dr.Eng.: Tokyo Institute of Technology (Currently, Professor Emeritus)

二羽淳一郎, 工学博士 : 東京工業大学 (現 名誉教授)

**Contact:** devin@kajima.com

**Keywords:** PC beam, shear capacity, shear-resisting mechanism, beam action, arch action

**DOI:** 10.11474/JPCI.NR.2022.205

## 1. Introduction

This study aimed to develop a calculation method for the shear capacity of prestressed concrete (PC) beams based on the shear-resisting mechanisms known as beam and arch actions. Static four-point bending tests were conducted on PC beams with various prestress levels and stirrup ratios, along with reinforced concrete (RC) beams. The contribution of each mechanism was calculated based on the strain of tensile steel bars, PC rods, and stirrups, and the effects of prestress level and stirrup ratio on the beam and arch actions were considered. Based on the results, a method for calculating the shear capacity of PC beams was developed. Comparison between experimental and analytical results shows that the developed method is more accurate for estimating the shear capacity of PC beams with stirrups.

## 2. Scope of the Study

### (1) Experimental Study

Static four-point bending tests were conducted on RC and PC beams. The prestress level at mid-depth  $\sigma_{cg}$  was 1.10–3.20 N/mm<sup>2</sup>, the stirrup ratio  $r_w$  was 0.00%–0.42%, the shear span to effective depth ratio was 3.8, and the concrete compressive strength was 46.4–53.0 N/mm<sup>2</sup>. The contributions of beam and arch actions at every shear force level were evaluated based on the relationship between shear force  $V$  and the distributions of tensile force  $T$  and lever arm length  $jd$  within the shear span, as shown in Eq. (1) [1]. Here,  $jd(dT/dx)$  expresses the contribution of the beam action ( $V_{beam}$ ), which is considered to include the resistance provided by the

truss mechanism of stirrups ( $V_{sbeam}$ ), and the resistance by concrete ( $V_{cbeam}$ ) in the form of mechanisms such as aggregate interlocking and dowel action, and  $T(d(jd)/dx)$  expresses the contribution of the arch action ( $V_{arch}$ ), which is formed by the inclined internal compression along the shear span. Thus, in this study, Eq. (2) was assumed. For PC beams, this relationship can be applied when the prestress is uniform along the span.

$$V = \frac{dM}{dx} = jd \frac{dT}{dx} + T \frac{d(jd)}{dx} \quad (1)$$

$$V = V_{beam} + V_{arch} = (V_{sbeam} + V_{cbeam}) + V_{arch} \quad (2)$$

The evaluation results of  $V_{sbeam}$ ,  $V_{cbeam}$ , and  $V_{arch}$  for specimens A ( $\sigma_{cg} = 1.16$  N/mm<sup>2</sup>,  $r_w = 0.21\%$ ) and B ( $\sigma_{cg} = 3.20$  N/mm<sup>2</sup>,  $r_w = 0.21\%$ ) are shown in **Fig. 1**. These were calculated using the strain distributions of tensile steel bars, PC rods, and stirrups measured during loading. It was shown that  $V_{cbeam}$  initially resisted most of the shear force, but it decreased after the initiation of diagonal cracks, with some portion of it remaining at failure. In contrast,  $V_{sbeam}$  started increasing after the initiation of diagonal cracks and eventually became flat, indicating the yielding of stirrups in the shear span.  $V_{arch}$  also started increasing after the initiation of diagonal cracks, and it continuously increased along with shear force  $V$  until failure.

### (2) Calculation of Shear Capacity

Based on the observed behaviors of the beam and arch actions, the authors developed a method for estimating

the contributions of  $V_{arch}$ ,  $V_{sbeam}$ , and  $V_{cbeam}$  at peak load, and their sum is taken as the shear capacity.

The shear capacity carried by stirrups in the beam action  $V_{sbeam\_cal}$  can be calculated using truss theory, assuming that the stirrups yield at failure.

The shear capacity carried by concrete in the beam action  $V_{cbeam\_cal}$  can be calculated by multiplying the diagonal crack initiation force (which is approximately  $V_{c\_cal}^{[2]}$ ) by a decrement factor  $\alpha$ ; this expresses the portion of  $V_{cbeam}$  maintained at failure. The decrement factor  $\alpha$  was found to decrease with higher prestress level  $\sigma_{cg}$  and stirrup amount  $r_w f_{wy}$ , as shown in Fig. 2.

The shear capacity carried by the arch action  $V_{arch\_cal}$  can be calculated by considering the compressive forces at a loading point ( $C_{LP}$ ) and a supporting point ( $C_{SP}$ ).  $C_{LP}$  is calculated from the compressive strength of concrete, while  $C_{SP}$  is calculated iteratively taking into account the moment shift due to diagonal cracks.

The calculation flow is summarized in Fig. 3, with some of the equations from design codes<sup>[2,3]</sup> and others from previous studies<sup>[4]</sup>. The calculation results were compared with experimental and analytical results for PC and RC beams obtained in this study and previous studies (see, for example, Watanabe et al.<sup>[5]</sup>). As shown in Fig. 4, the developed calculation method (Proposed method) gives a better estimation for shear capacity compared with the conventional method using modified truss theory (MTT)<sup>[2]</sup>.

### 3. Conclusion

- Within a considered range, the shear capacity carried by stirrups in the beam action can be calculated using truss theory.
- The shear capacity carried by concrete in the beam action can be calculated by applying a decrement factor to the diagonal crack initiation force.
- The shear capacity carried by the arch action can be calculated by considering the compression forces at loading and supporting points.
- The developed method shows better accuracy for estimating the shear capacity of RC and PC beams with stirrups.

### References

[1] Park, R., Paulay, T.: *Reinforced Concrete Structures*, John Wiley & Sons, pp.278-287, Jul. 1975.  
 [2] JSCE: *Standard Specifications for Concrete Structure-2017“design”*, Japan Society of Civil Engineers, pp.430-432, Mar. 2017 (in Japanese).  
 [3] fib: *fib Model Code for Concrete Structures-2010*, International Federation for Structural Concrete, p.235, Oct. 2013.  
 [4] Sato, Y. et al.: *Shear Strength of Prestressed Concrete Beams with FRP Tendon*, Proceedings of JSCE, No. 520/V-28, Japan Society of Civil Engineers, pp.213-224, Aug. 1995 (in Japanese).  
 [5] Watanabe, H. et al.: *Shear Strength of PC Beams with High Strength Lightweight Aggregate Concrete*, Concrete Research and Technology, No.1, Vol.14, Japan Concrete Institute, pp.13-22, Jan. 2003 (in Japanese).

doi:10.2749/101686611X13049248220276

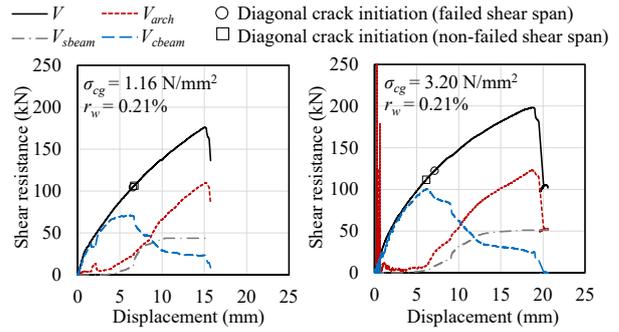


Fig. 1 Transition of shear-resisting mechanism

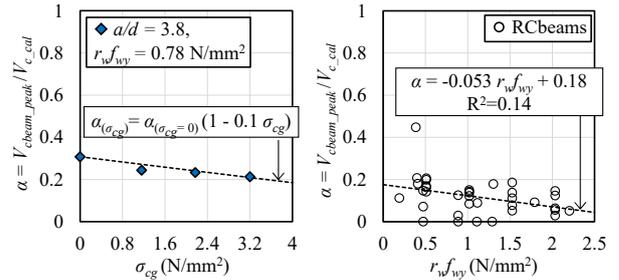


Fig. 2 Variation of decrement factor  $\alpha$

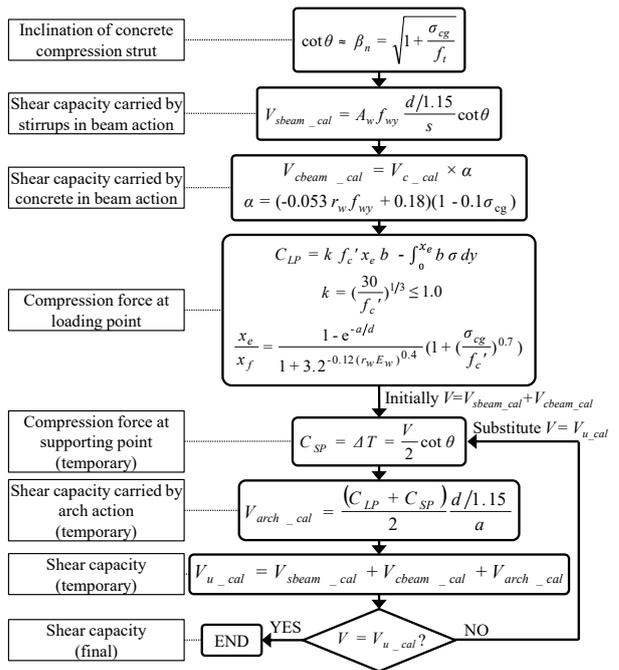


Fig. 3 Calculation flow for shear capacity

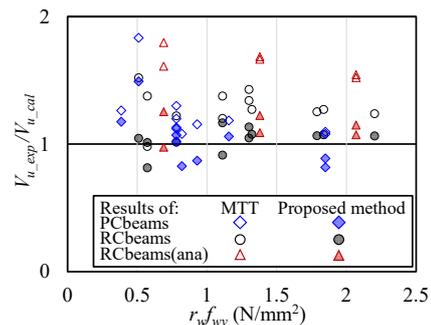


Fig. 4 Comparison of estimated shear capacity